

Electroweak baryon number non-conservation and “topological condensates” in the early Universe

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Electroweak vacuum transition processes (sphalerons) in the early Universe provide a possible explanation of the baryon asymmetry. Anomaly theory suggests that these electroweak baryon number non-conserving processes are accompanied by the formation of a “topological condensate” which may remain in the Universe that we observe today.

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1 Introduction

Understanding the baryon asymmetry of the observed Universe is one of the key challenging problems at the interface of particle physics and cosmology. Sakharov identified three famous conditions which must be satisfied in any theory which claims to explain the physics [1]:

1. Baryon number non-conservation
2. CP and C violation so that processes occur at different rates for particles and antiparticles
3. Departure from thermal equilibrium. Otherwise if the Universe starts with zero baryon number it will stay with zero baryon number.

In this paper we focus on the first condition. Non-perturbative electroweak baryogenesis offers the intriguing possibility that topological features of the Standard Model might be (in part) responsible for baryon and lepton number violation in the early Universe [2, 3, 4, 5].

In this talk we argue that electroweak baryogenesis in the early Universe is accompanied by the formation of a “topological condensate” in the Standard Model vacuum [6] which (probably) survives cooling to the Universe that we observe today. The key physics involves the application of anomalous commutator theory [7, 8, 9] and renormalization group arguments to non-perturbative vacuum transition processes (sphalerons) between vacuum states characterized by different topological winding numbers. When the effect of QCD vacuum transition processes which break axial U(1) symmetry are also included the net “topological condensate” develops a spin independent component.

We first outline the key features of electroweak baryogenesis and then explain the consequences of anomalous commutator theory for this physics. The existence (or otherwise) of the “topological condensate” will depend on the precise definition

of baryon number in the presence of the axial anomaly and how to include non-local structure associated with gauge field topology into local anomalous Ward Identities.

In the Standard Model the parity violating SU(2) electroweak interaction induces an axial anomaly contribution [10] in the (vector) baryon number current [11]. Through electroweak instantons this leads to the possibility of baryon (and lepton) number violation through quantum tunneling processes in the θ -vacuum [12] for the Standard Model fields. This baryon number violation never appears in perturbative calculations but is generated through nonperturbative transitions between different vacuum states. Each transition violates baryon number (and lepton number) by $\Delta B = \Delta L = \pm 3n_f$ where n_f is the number of families (or fermion generations). For example, for $n_f = 3$, we find electroweak processes such as

$$q + q \rightarrow 7\bar{q} + 3\bar{l} \quad (1)$$

where all the fermions in this equation are understood to be left handed and there are 3 quarks and one lepton from each generation. At zero temperature these transitions are exponentially suppressed by the factor

$$e^{-4\pi \sin^2 \theta_W / \alpha} \sim 10^{-170} \quad (2)$$

and are therefore negligible. Kuzmin, Rubakov and Shaposhnikov [3] argued that this baryon number violating process becomes unsuppressed at high temperature $T \gg M_W$, and is thus a candidate for baryon number violation in the early Universe. The reason that the suppression goes away is that the transition can then arise due to thermal fluctuations rather than quantum tunneling once the temperature becomes high compared to the potential barrier V_0 between the different vacuum states. Electroweak vacuum transitions involve (just) left-handed fermions through the parity violating couplings of the electroweak SU(2) gauge fields. Additional QCD sphaleron transition processes (mediated through the strong QCD axial anomaly) plus couplings to scalar Higgs field(s) offer possible mechanisms for transfer of the baryon number violation to right handed quarks [2].

The anomalous electroweak baryon number violating process might also be observable in (very) high energy proton-proton collisions when the centre of mass energy in the parton parton collision exceeds the potential barrier between the different vacuum states [13]. The energies involved are very large and will most likely require an accelerator like the VLHC with several hundred TeV in energy [14].

2 What is baryon number ? The axial anomaly and anomalous commutators

The definition of baryon number in quantum field theories like the Standard Model is quite subtle because of the axial anomaly.

The vector baryon current can be written as the sum of left and right handed currents:

$$J_\mu = \bar{\Psi} \gamma_\mu \Psi = \bar{\Psi} \gamma_\mu \frac{1}{2} (1 - \gamma_5) \Psi + \bar{\Psi} \gamma_\mu \frac{1}{2} (1 + \gamma_5) \Psi. \quad (3)$$

Classically this fermion current is conserved. In quantum field theories the current must be regularized and renormalized. In the Standard Model the left handed fermions couple to the SU(2) electroweak gauge fields W and Z^0 . As 't Hooft first pointed out [11], this means that this baryon current is sensitive to the axial anomaly. One finds the anomalous divergence equation

$$\partial^\mu J_\mu = n_f \left(-\partial^\mu K_\mu + \partial^\mu k_\mu \right) \quad (4)$$

where K_μ and k_μ are the SU(2) electroweak and U(1) hypercharge Chern-Simons currents

$$K_\mu = \frac{g^2}{16\pi^2} \epsilon_{\mu\nu\rho\sigma} \left[A_a^\nu \left(\partial^\rho A_a^\sigma - \frac{1}{3} g f_{abc} A_b^\rho A_c^\sigma \right) \right] \quad (5)$$

and

$$k_\mu = \frac{g'^2}{16\pi^2} \epsilon_{\mu\nu\rho\sigma} B^\nu \partial^\rho B^\sigma. \quad (6)$$

Here A_μ and B_μ denote the SU(2) and U(1) gauge fields, and $\partial^\mu K_\mu = \frac{g^2}{32\pi^2} W_{\mu\nu} \tilde{W}^{\mu\nu}$ and $\partial^\mu k_\mu = \frac{g'^2}{32\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu}$ are the SU(2) and U(1) topological charge densities. Eq.(4) allows us to define a conserved current

$$J_\mu^{\text{con}} = J_\mu - n_f (-K_\mu + k_\mu). \quad (7)$$

The current J_μ^{con} satisfies the divergence equation

$$\partial^\mu J_\mu^{\text{con}} = 0 \quad (8)$$

but is SU(2) and U(1) gauge dependent because of the gauge dependence of K_μ and k_μ . When we make a gauge transformation U the SU(2) electroweak gauge field transforms as

$$A_\mu \rightarrow U A_\mu U^{-1} + \frac{i}{g} (\partial_\mu U) U^{-1} \quad (9)$$

and the operator K_μ transforms as

$$K_\mu \rightarrow K_\mu + i \frac{g}{8\pi^2} \epsilon_{\mu\nu\alpha\beta} \partial^\nu \left(U^\dagger \partial^\alpha U A^\beta \right) + \frac{1}{24\pi^2} \epsilon_{\mu\nu\alpha\beta} \left[(U^\dagger \partial^\nu U) (U^\dagger \partial^\alpha U) (U^\dagger \partial^\beta U) \right] \quad (10)$$

where the third term on the RHS is associated with the gauge field topology [15]. Because of the absence of topological structure in the U(1) sector it is sufficient to drop the U(1) “ k_μ contribution” in discussions of anomalous baryon number violation, which we do in all discussion below. Conserved and partially conserved currents are not renormalized. It follows that J_μ^{con} is renormalization scale invariant. The gauge invariantly renormalized current J_μ is scale dependent with the two-loop anomalous dimension induced by the axial anomaly – the scale dependence of J_μ is carried entirely by K_μ [10, 15].

Equation (4) presents us with two candidate currents we might try to use to define the baryon number: J_μ and J_μ^{con} . We next explain how both currents yield

gauge invariant possible definitions. The selection which current to use has interesting physical consequences which we discuss in Section 3. We shall require our definition of baryon number to be renormalization scale invariant and that the W boson fields and their time derivatives carry zero baryon number.

We choose the A_0 (and $B_0 = 0$) gauge and define two operator charges:

$$Y(t) = \int d^3z J_0(z), \quad B = \int d^3z J_0^{\text{con}}(z). \quad (11)$$

Because conserved currents are not renormalized it follows that B is a time independent operator. The charge $Y(t)$ is manifestly gauge invariant whereas B is invariant only under “small” gauge transformations; the charge B transforms as

$$B \rightarrow B + n_f m \quad (12)$$

where m is the winding number associated with the gauge transformation U . Although B is gauge dependent we can define a gauge invariant baryon number \mathcal{B} for a given operator \mathcal{O} through the gauge-invariant eigenvalues of the equal-time commutator

$$[B, \mathcal{O}]_- = \mathcal{B} \mathcal{O}. \quad (13)$$

(The gauge invariance of \mathcal{B} follows since this commutator appears in gauge invariant Ward Identities [15] despite the gauge dependence of B .) The time derivative of spatial components of the W-boson field have zero baryon number \mathcal{B} but non-zero Y charge:

$$[B, \partial_0 A_i]_- = 0 \quad (14)$$

and

$$\lim_{t' \rightarrow t} \left[Y(t'), \partial_0 A_i(\vec{x}, t) \right]_- = \frac{in_f g^2}{4\pi^2} \tilde{W}_{0i} + O(g^4 \ln |t' - t|) \quad (15)$$

with $\tilde{W}_{\mu\nu}$ the SU(2) dual field tensor. (See Refs.[7, 8, 15] for a discussion of the analogous situation in QED and QCD.) Eq.(14) follows from the non-renormalization of the conserved current J_μ^{con} . Eq.(15) follows from the implicit A_μ dependence of the (anomalous) gauge invariant current J_μ . The higher-order terms $g^4 \ln |t' - t|$ are caused by wavefunction renormalization of J_μ [15].

Motivated by this discussion of anomalous commutators, plus the renormalization scale invariance of baryon number, we next choose to identify baryon (and lepton) number with the gauge invariant commutators of the charge B associated with conserved current J_μ^{con} , Eq.(13). We investigate the physical consequences of this choice ¹⁾ and compare our results with the physics obtained if one instead uses the gauge invariantly renormalized current J_μ and the charge $Y(t)$ to define the “baryon number”.

¹⁾ Traditionally, the electroweak baryogenesis literature has implicitly assumed that baryon and lepton number are associated with eigenvalues of the charge Y of the gauge-invariantly renormalized current J_μ . We believe that the arguments presented above imply that the conserved current definition presents at the minimum a legitimate alternative whose physical consequences should be explored.

3 Gauge topology and vacuum transition processes

When topological effects are taken into account, the Standard Model vacuum $|\theta\rangle$ is a coherent superposition

$$|\theta\rangle = \sum_m e^{im\theta} |m\rangle_{\text{EW}} \quad (16)$$

of the eigenstates $|m\rangle_{\text{EW}}$ of $\int d\sigma_\mu K^\mu \neq 0$ [15]. Here σ_μ is a large surface which is defined such that its boundary is spacelike with respect to the positions z_k of any operators or fields in the physical problem under discussion. For integer values of the topological winding number m , the states $|m\rangle_{\text{EW}}$ contain $4mn_f$ fermions (3 quarks and one lepton from each fermion generation) carrying baryon and lepton number $B = L = n_f \xi_{\text{EW}}$ (and zero net electric charge). The factor ξ_{EW} is equal to +1 if the baryon number is associated with J_μ^{con} and equal to -1 if the baryon number is associated with J_μ — see below. Relative to the $|m = 0\rangle_{\text{EW}}$ state, the $|m = +1\rangle_{\text{EW}}$ state carries electroweak topological winding number +1 and $3n_f$ quarks and n_f leptons with baryon and lepton number $B = L = n_f \xi_{\text{EW}}$.

Following from Eqs.(4) and (7), in electroweak sphaleron (or instanton) induced vacuum transition processes

$$\Delta Y = \Delta B - n_f m \quad (17)$$

where $m = \pm 1$ is the change in the electroweak topological winding number. The change in winding number is an integer for these processes and renormalization scale independent. The anomalous commutators (14,15) and renormalization group invariance suggest that we associate the change in the baryon number with the baryonic charge B in this equation: $\Delta B = n_f m$ with $\Delta Y = 0$.

We now consider the physical effect of the choice of baryon number current. For the sake of definiteness we consider a vacuum transition characterized by a change in the electroweak topological winding number $m = +1$.

1. First consider the scenario where the baryon number is associated with the conserved vector current J_μ^{con} through the gauge invariant commutators of the charge B , Eq.(13). Here $\Delta B = n_f$ and $\Delta Y = 0$ in the sphaleron transition process. Energy and momentum are conserved between the particles which are produced and absorbed in the non-perturbative transition, eg. Eq.(1), which produces the baryon and lepton number violation. The topological term coupled to K_μ which measures the change in the winding number (or change in the gauge-field boundary conditions at infinity) carries zero energy and zero momentum. Thus, the change in the baryon number ΔB is compensated by a shift of quantum-numbers with equal magnitude but opposite sign into the “vacuum” (defined here as everything carrying zero four-momentum) so that Y is conserved. In this scenario the “vacuum” acquires a “topological charge” which compensates the baryon and lepton number non-conservation (plus chirality since electroweak sphalerons/baryogenesis act just on left-handed fermions).

2. In the alternative scenario where baryon number is identified with the current J_μ the non-conserved “baryon number” is identified with Y in Eq.(17), viz. $\Delta Y = -n_f$ and $\Delta B = 0$. It is illuminating to consider the anatomy of this process, looking at the details needed to restore B conservation. For QCD instantons this was discussed by ’t Hooft – see Section 6 of his paper [16]. An effective “schizon” object needs to be introduced to absorb in the “vacuum” B quantum numbers equal in magnitude and opposite in sign to those induced by the change in the topological winding number. The “schizon” carries zero energy and zero momentum and acts to cancel the zero-mode contribution obtained in the previous “ J_μ^{con} baryon number” scenario. The “schizon” is constructed to produce a theory with no net transfer of quanta to the “vacuum” under instanton or sphaleron transition processes. This contrasts with the first scenario where the vacuum does acquire net quantum numbers since the total Y charge is conserved. (Note also the difference in the sign of the baryon number violation induced with the two definitions.)

What is the practical effect of using the charge B to define baryon number ?

In this scenario the total charge Y and the information measured by it are conserved in electroweak vacuum transitions: the “vacuum” will acquire topological quantum numbers equal to minus the change in baryon (and lepton) number and minus the change in (left-handed) chirality induced by electroweak sphalerons. That is, this electroweak baryogenesis process is accompanied by the formation of a “topological condensate” in the “vacuum” carrying these quantum numbers.

QCD sphalerons will also be at work in high temperature processes and (together with Higgs couplings) act to distribute the baryon number violation between both left and right handed quarks [2]. The same arguments that we used for baryon number violation generalize readily to axial U(1) violation in QCD instanton/sphaleron processes [15, 16, 17]. One finds [6] QCD sphalerons act to cancel the chirality dependence of the (quark part of the) electroweak vacuum generated by the electroweak sphalerons. QCD sphaleron processes which shift the baryon number violation from left to right handed quarks also act to cancel the chiral polarization of the “vacuum” induced by electroweak sphalerons. Hence, the result is to create a spin/chiral independent component in the net “topological condensate” which forms in electroweak baryogenesis (corresponding to finite baryon number violation with the chiral dependence, at least in part, cancelled.)

The QCD analogue of this physics has been studied in the context of the proton spin [19] and axial U(1) [15] problems. For the proton spin problem, gluon topological effects have the potential to induce a “subtraction at infinity” correction to the Ellis-Jaffe sum-rule for polarized deep inelastic scattering [18] associated with the circle at infinity when we close the contour in the dispersion relation for polarized photon nucleon scattering. This correction, if finite, corresponds to a Bjorken $x = 0$ (or “polarized condensate” [17]) contribution to the nucleon’s flavour-singlet axial charge $g_A^{(0)}$ generated through dynamical axial U(1) symmetry breaking. A direct measurement of $g_A^{(0)}$, independent of this possible correction, could be obtained from a precision measurement of elastic νp scattering [20].

4 Conclusions

Renormalization group invariance and anomalous commutator theory suggests that sphaleron induced electroweak baryogenesis in the early Universe is accompanied by the formation of a “topological condensate”. QCD sphalerons tend to cancel the spin dependence of the quark part of this “condensate” leaving the baryon number violating component untouched. It seems reasonable to postulate that this “topological condensate” survives the cooling to present times along with the baryon number violation induced by baryogenesis.

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